Medical Physics

Ch4 : Dynamics

12/Nov/2024

Medical Physics

* Dynamics : Newton's Laws of motion

- Force : is any Kind of Pull or Push acting on an object

Newton's Laws of motion :

*** First Law :** Every objects continue in its state of rest or of a uniform velocity in a straight line as long as there is NO net force acting on it
[Law of inertia]

- There is a diffrence between mass & weight

mass : is the inertia of an object (resistance to change in its state of motion) and its measured in (Kg) SI system

while weight is : the force of gravity on an object
($W = mg$)

*** Secon law :** acceleration is directly Proportional to the net force on it and Inversly Proportional to the mass of the object

$$\boxed{\sum F = ma}$$

- $\sum \vec{F} = m\vec{a}$
- $\sum F_y = mg$
- $\sum F_x = m a_x$

the unit of Force is (Newton) (N)
since :
$F = ma$
$F = Kg \cdot \frac{m}{s^2}$
$N = Kg \cdot m/s^2$

examples

estimate the net force needed to accelerate:

- a 1000kg car at $\frac{1}{2}g$

$\Sigma F = ma$

$F = 1000\left(\frac{1}{2}(9.8)\right) \simeq 5000\,N$

... and so on

Third Law: each action has a reaction that is equal in magnitude & in opposite direction

$$F_{12} = -F_{21}$$

F_{12} ↳ the force that 1 exerts on 2

F_{21} ↳ the force that 2 exerts on 1

---

weight - The force of gravity & the Normal force

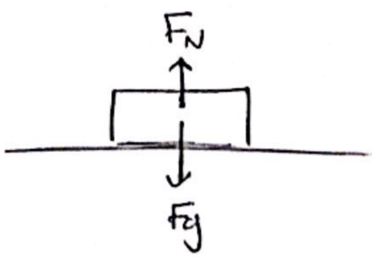
* as we know the gravitational force on an object is

$$\Sigma \vec{F}_G = m\vec{g}$$

and when there is a contact between an object and a surface there is a force that is in opposite of the gravitational force and perpindicular to the surface and referred to as <u>Normal force</u> (F_N)

- in equilibrium (on a flat surface horizatally):

$$F_N = F_G$$

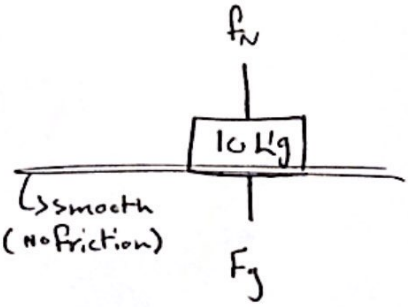


Just like the Third Law of motion

example:

box : mass = 10 kg

1) weight??
Normal force??



weight = mg

$F_g = 10 \times \sim 10 = \underline{100\ N}$

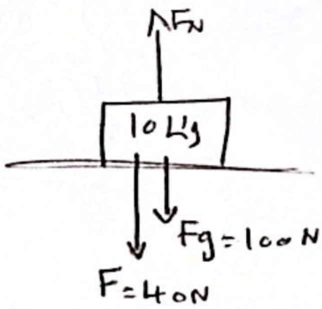
on a horizontal surface and the system is in equilibrium
اتزان
↳ meaning Net Force = 0

$\Sigma F = 0$

$F_N - F_g = 0$
↳ bc its downwards (−y)

$F_N = F_g = 100\ N$

2) if we push on the box down with a 40 N force whats the F_N

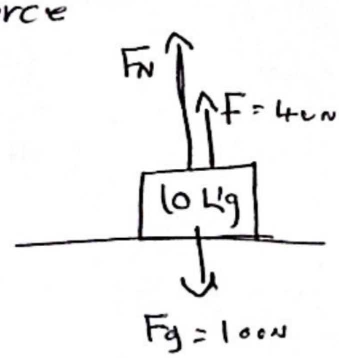


$\Sigma F = 0$

$F_N - F - F_g = 0$

$F_N = F + F_g = \underline{140\ N}$

3) if we pull the box upwards with 40 N force whats F_N then:



$\Sigma F = 0$

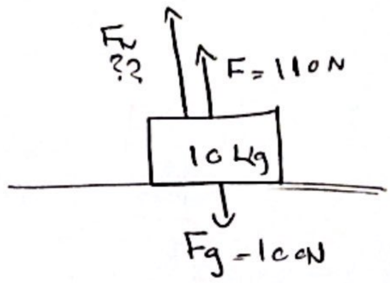
$F_N + F - F_g = 0$
bc its up (+y)

$F_N = F_g - F$

$F_N = 60\ N$

④ what happens when we pull upward on the box with a force equal or greater than the box weight

So ex: 110N force



this means:

if $\Sigma F = 0$

$F_N + F - F_g = 0$

$F_N = F_g - F$

$F_N = 100 - 110 = -10\ N$

↳ but this is impossible bc the least magnitude of a force can be = 0

This means that there is acceleration and the box is <u>NOT</u> in equilibrium anymore

$\Sigma F_y = m\, a_y$

$F_y = F - F_g$

$= 110 - 100 = \underline{10N}$

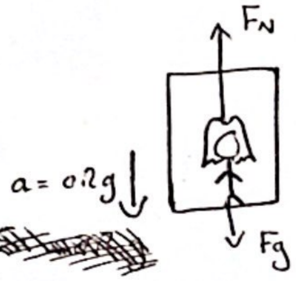
$a_y = \frac{F_y}{m} = \frac{10}{10} = \underline{1\ m/s^2}$

ex:

woman
$m = 65\ kg$
in a descending elevator (مصعد نازل)
at an acceleration of 0.2g
↳ gravity acc
and she steps on a scale

① whats her weight as the elevator descends and what does the scale read

$F_g = mg = 65 \cdot (9.8) = 637N = F_g$



since this is NOT at equilibrium
(there is an acceleration)

$\Sigma F = ma$

$F_N - F_g = ma$ ↳ be going down
$F_N - 637 = 65\ (0.2g)$
$F_N = 509.6\ N$

here the scale will show that her mass is $509.6 = m\ (9.8)$
$m = 52\ kg$
but her actuall mass didn't change

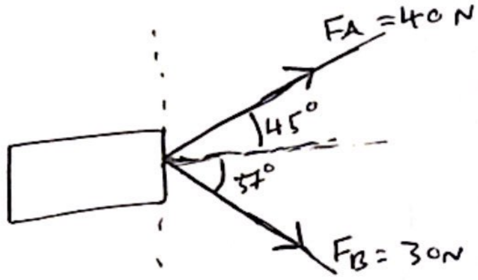
② what if the elevator was descending at a constant speed of $2\ m/s$
↳ means $a = 0\ m/s^2$
means No force acting
(at equilibrium)
$\Sigma F = 0$
bc
$\Sigma F = ma$
$\Sigma F = m(0)$
$\Sigma F = 0$

$\Sigma F = 0$

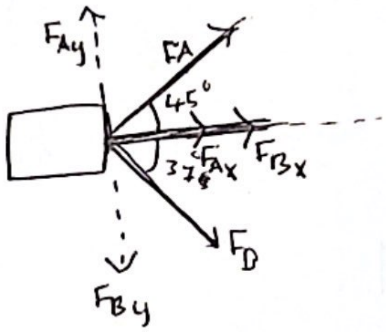
$F_N = F_g = 637N$

using free-body diagrams

ex: Calculate the Sum (Σ (علامة)) of the forces :



. . . → splitting to components :



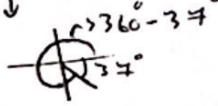
$R_y = F_{Ay} + F_{By}$

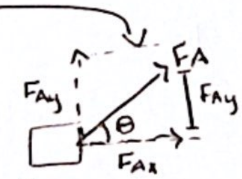
$F_{Ay} = F_A \sin\theta = 28.28\,N$ ($\theta \to 45°$)

$F_{By} = F_B \sin\theta = -18.05\,N$ ($\theta \to -37°$)

→ we put (−) bc we went clockwise and the original way of doing it is taking the angle from the (+x) axis counter clockwise

so $\sin(-37°) = \sin(360° - 37°)$





$\sin\theta = \frac{F_{Ay}}{F_A}$

$F_{Ay} = F_A \sin\theta$

(same for B)

$R_y = 10.2\,N$

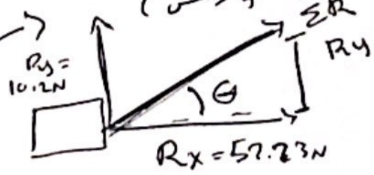
$R_x = F_{Ax} + F_{Bx}$

$F_{Ax} = F_A \cos\theta = 28.28\,N$

$F_{Bx} = F_B \cos\theta = 23.9\,N$

$R_x = 52.23\,N$

Pythgreo & theorem (فيثاغورس)



$\Sigma R^2 = R_y^2 + R_x^2$

$\Sigma R = \sqrt{(R_x)^2 + (R_y)^2}$

$\Sigma R = \sqrt{(10.2)^2 + (52.23)^2}$

$\Sigma R = 53.2\ N$

---

to know the direction of force :

we use $\tan\theta$

$\tan\theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

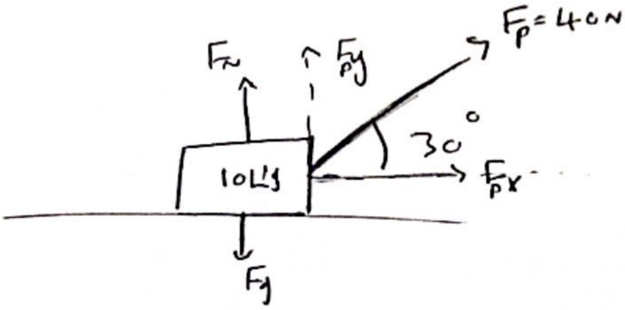
$\theta = \tan^{-1}\left(\frac{10.2}{52.2}\right) = 11.05°$ from the x-axis

Q: 4-11

box:

m = 10 Kg

Pulled with a Rope on a smooth surface with a force of 40N and 30° angle



① calculate the acceleration:

$F_x = ma_x$

↳ since there is No movement up and down

$F_{px} = F_p \cos\theta = 34.6N$

$F_x = ma_x$

$34.6 = 10\, a_x$

$a_x = 3.46\ m/s^2$

② Calculate F_N

$F_{py} = F_p \sin\theta = 20N$

$\Sigma F = 0$

$F_g = mg = 100N$

$F_N + F_{py} = F_g$

$F_N = 100 - 20 = 80N$

④

ex 4-12

boxes A & B connected with a cord and resting on a smooth surface

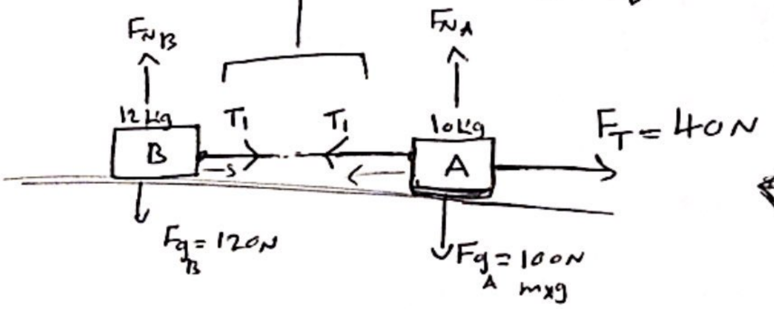
A's mass is 10 kg

B's mass is 12 kg

a force (horizontally) applied to pull A with 40 N

- find the acceleration of each box
- find the tension in the connecting cord

→ the tension forces in the cord are equal and in opposite direction (the third law)



① we draw the free-body diagrams

$\Sigma F_{A_x} = m_a a_x$

$\Sigma F_{B_x} = m_b a_x$

→ both of the boxes have the same acceleration bc if not they would move either far apart or closer and hit each other so they have to have the same acceleration

so: for $\Sigma F_{Ax} = m_A a_x$

$F_T - T_1 = m_A a_x$ ...①

made formulas

for $\Sigma F_{Bx} = m_B a_x$

$T_1 = m_B a_x$ ...②

→ using this equation:

$a_x = \frac{T_1}{m_B}$

→ plugging into eq(1)

$F_T - T_1 = m_A \cdot \frac{T_1}{m_B}$

→ direction (-x)

$\frac{40 - T_1}{1} \times \frac{10 \cdot T_1}{12}$

$12(40 - T_1) = 10T_1$

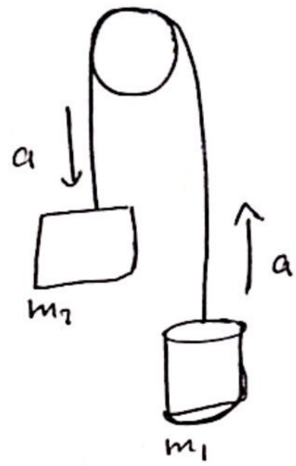
$40 - T_1 = \frac{5}{6} T_1$

$40 = \frac{11T_1}{6} \Rightarrow T_1 = 21.8\ N$

replug it

$a_x = \frac{21.8}{12} = 1.818\ m/s^2$

The atwood machine:

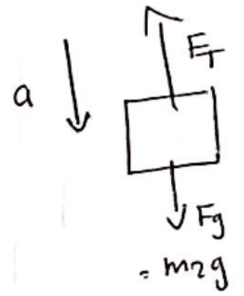


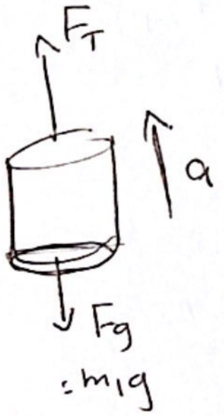
① Calculate the acceleration of the elevator:

they both have the same acceleration bc if not they would collide or not work

- first we draw free-body diagrams

$m_1 = 1000\,kg$
$m_2 = 1100\,kg$





So:
we make equations:

$\Sigma F \neq 0$ bc its moving in acceleration

so $\Sigma F = ma$

for m_1: $F_T - F_g = m_1 a_y$

$F_T - m_1 g = m_1 a_y$

$F_T = m_1 a_y + m_1 g$ ... ①

for m_2: $F_T - F_g = (-) m_2 a_y$ (bc a is in (−) direction (−y))

Plug in: $F_T - m_2 g = -m_2 a_y$ ... 2

$m_1 a_y + m_1 g - m_2 g = -m_2 a_y$

$m_1 g - m_2 g = -m_2 a_y - m_1 a_y$

$g(m_1 - m_2) = -a_y(m_2 + m_1)$

$a_y = -\dfrac{g(m_1 - m_2)}{m_2 + m_1} = \dfrac{g(m_2 - m_1)}{m_2 + m_1} = \dfrac{10(1100 - 1000)}{1100 + 1000} = 0.47\,m/s^2$

② find the tension:

we use: $F_T = m_1 a_y + m_1 g$

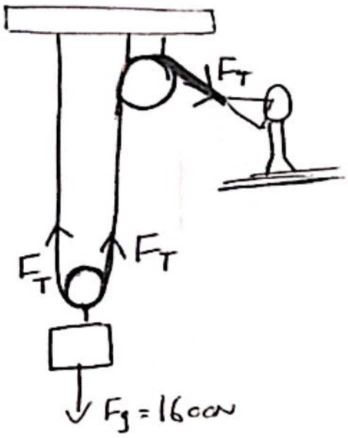
$F_T = m_1(a_y + g)$

$F_T = 1000(0.47 + 10)$

$F_T = 10470\,N$

lifting a weight using a rope looped around 2 Pullys

- what force that should be exert on the rope to lift the weight which is 1600N with a constant speed ($a=0$)



$$\Sigma F = 0$$

$$2F_T - F_g = 0$$

$$F_T = \frac{F_g}{2}$$

$$F_T = \frac{1600}{2} = 800N$$

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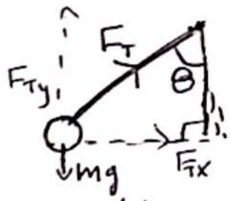
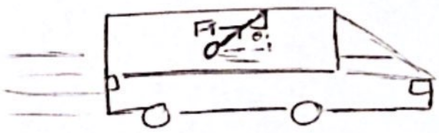
a small mass (m) hanging and can swing like a Pendulum

- what angle θ does the string make when:

① when the car accelerates at a constant $a = 1.2\ m/s^2$

make a free body diagram:





Since the is acceleration $\Sigma F \neq 0$ in the x-axis

[illegible]

So we split F_T to its components and work on 2 axis x & y:

x: $\Sigma F = ma_x$

$F_{Tx} = ma_x$

y: $\Sigma F = 0$ ~ No movement on y

$F_{Ty} - mg = 0$

$F_{Ty} = mg$

to get θ: $\tan\theta = \frac{F_{Tx}}{F_{Ty}} = \frac{ma_x}{mg}$

$$\theta = \tan^{-1}\left(\frac{ma_x}{mg}\right)$$

$$\theta \approx 7^\circ$$

---

② when the car is moving at a constant velocity = 90 Km/h

x: $F_{Tx} = 0$

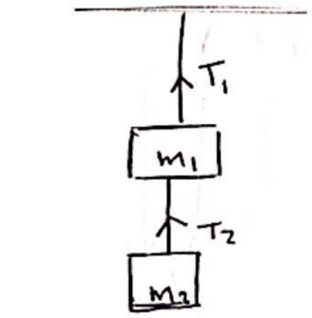
y: $F_{Ty} = mg$

$$\theta = \tan^{-1}\left(\frac{0}{mg}\right)$$

$$\theta = \tan^{-1}(0) = 0^\circ$$

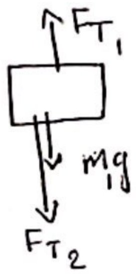
*exercise:

Two masses hang by two cords as shown
- find T_1 & T_2

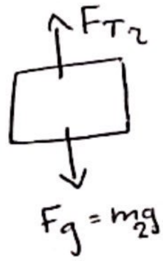


$m_1 = m_2 = 5$

For m_1



For m_2



$\Sigma F = 0 \qquad F_{T_1} = m_1 g + F_{T_2}$ ← $F_{T_2} = m_2 g$

$F_{T_1} = m_1 g + m_2 g$

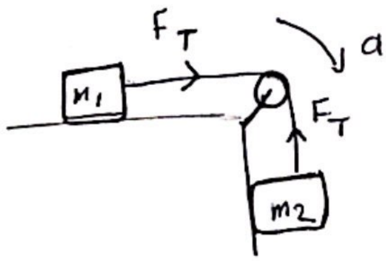
$F_{T_1} = g(m_1 + m_2)$

$\boxed{F_{T_1} = 10(10) = 100N}$

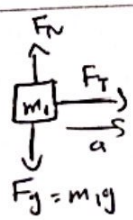
$\boxed{F_{T_2} = 50N}$

Exercise 2

find a & F_T:



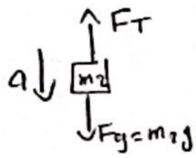
they would have the same acceleration



$\Sigma F \neq 0$

$\Sigma F_x = ma_x$

$F_T = m_1 a \quad \ldots (1)$



$\Sigma F_y \neq 0$

$F_T - m_2 g = m_2(-a) \quad \ldots (2)$

↓

$m_1 a - m_2 g = -m_2 a$

$m_1 a + m_2 a = m_2 g$

$a(m_1 + m_2) = m_2 g$

$\boxed{a = \frac{m_2 g}{m_1 + m_2}}$

so

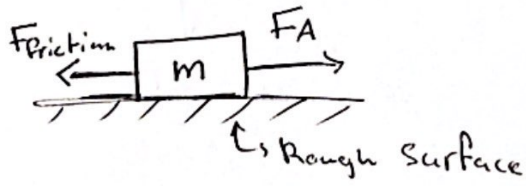
$F_T = m_1 \frac{(m_2 g)}{(m_1 + m_2)}$

* Problems involving Friction & inclines

- Friction -

- Consider a mass (m) at rest on a rough surface and a force (F_A) is trying to move the mass, but it remains at rest
  - ↳ why??
    - ↳ because of Friction

(It works in the opposite direction of the force on a rough surface)



and using Newton's second Law

$\sum F = 0$ → bc we said that the mass is still at rest

$F_A - F_{fric} = 0$ so ⇒ $\boxed{F_A = F_{fric}}$

and this always true as long as the mass is stationary

this is called the [static friction] (when the obj doesn't move)

and as soon as it moves its called [Kinetic friction]

their equations are:

$f_s \le \mu_s F_N$ → Normal Force

(f_s: stationary; μ_s ↳ coefficient of friction)

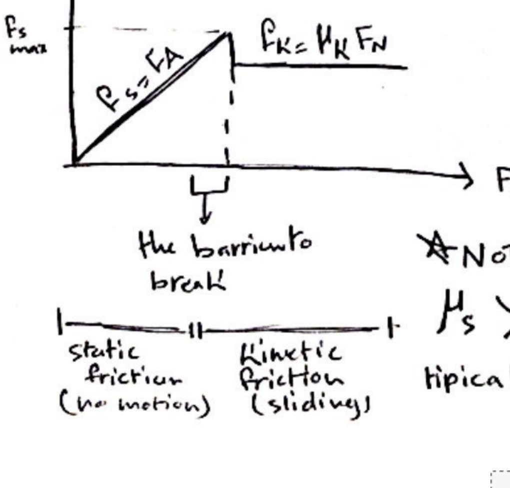
- while the $\le$ sign is used bc it shows that f_s is changing (Not a fixed amount) and you have to reach and break a certain barrier to move the object which will become easier to keep moving and you have to break that barrier every time you want to move the obj from static

while f_K:

$f_K = \mu_K \cdot F_N$ ↳ means fixed amount

(K ↳ Kinetic)

we use F_N bc its the actual force that presses the surfaces together, imagine an incline, the weight isn't the whole contributor to the force + the normal force is perpendicular to the surface



★ Note that: $\mu_s > \mu_K$ tipically ★

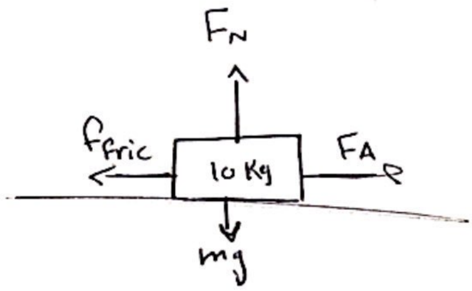
Example:

- A box of mass 10 Kg resting on a horizontal floor

$\mu_s = 0.4$ find the force of friction
$\mu_k = 0.3$ if a force (FA) is exerted on it
that has a magnitude of:

(a) 0 (b) 10N (c) 20N

(d) 39N (e) 41N



finding the forces both on the y and x axes

$\Sigma F_y = 0$ → No movement on y axis

$F_N = F_g = \underline{100N}$

→ $\Sigma F_x =$ → 0
→ or max

we don't know if its moving or not
and to know that:
we find the f_s barrier limit or the f_s max
which is

$f_{s_{max}} = \mu_s F_N$

$f_{s_{max}} = 40N$

→ so any force applied less than 40 the obj will **NOT** move
and that exactly when

$F_s = F_A$

(the f_s will always match the applied force to cancel it until the barrier is broken

(a) when applying 0N, $\Sigma F = 0$ means $f_s = f_A = \underline{0N}$

(b) 10N, less than 40N
$f_s = f_A = \underline{10N}$

(c) 20N, $f_s = f_A = \underline{20N}$

(d) 39N, $f_s = f_A = \underline{39N}$

(e) 41N > 40N ($f_{s\,max}$).
so the obj moves and now $f_s \to f_k$

$f_k = \mu_k \cdot F_N = \underline{30N}$

$\Sigma F_x = m a_x$

$F_A - F_k = 10 (a_x)$

$41 - 30 = 10\, a_x$

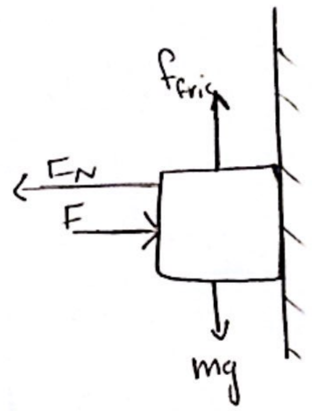
$a_x = 1.1\ m/s^2$

Ex: you can hold a box against a rough wall by pressing hard horizontally

$\mu_s = 0.4$

$mg = 20N$

what minimum force will keep the box from falling?



$\Sigma F_y = 0$

$f_s = mg = 20N$

$\Sigma F_x = 0$

$F_N = F$

$f_s \leq \mu_s F_N$

$f_s \leq \mu_s F$

$F_{min} > \frac{f_s}{\mu_s} \quad \frac{20}{0.4} = 50N$

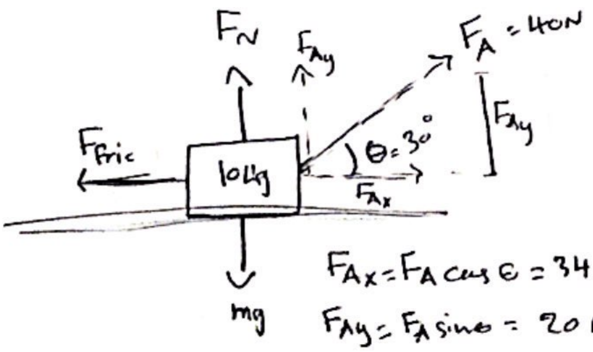
Ex: (Pulling against friction)

box, $m = 10kg$

Pulled on a horizontal surface with a 40N force at 30°

$\mu_k = 0.3$

calculate the acceleration



$F_{Ax} = F_A \cos\theta = 34.6N$

$F_{Ay} = F_A \sin\theta = 20N$

$\Sigma F_y = 0$

~~$F_N - F_g = 100N$~~

~~$\Sigma F_x = ma_x$~~

$F_N + F_{Ay} - F_g = 0$

$F_N = F_g - F_{Ay} = 100 - 20 = \boxed{80N}$

$\Sigma F_x = ma_x$

$F_{Ax} - F_{fric_k} = ma_x$

$34.6 - 24 = 10 a_x$

$a_x = 1.06 \ m/s^2$

$f_k = \mu_k F_N$

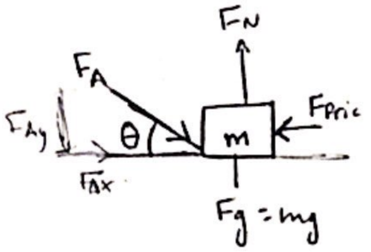
$= 0.3 \times 80$

$= 24N$

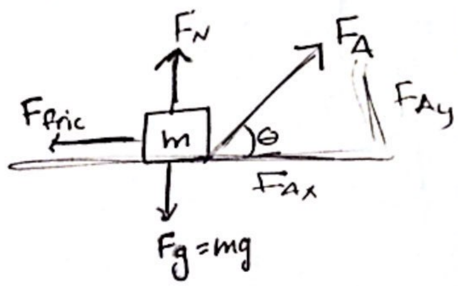
Pushing or pulling on a sled :

if you are wanting to move an object by sliding it will you exert ~~no~~ Less ~~friction~~ force by Pushing the obj or Pulling on it ??

Push :



Pull :



$\Sigma F_y = 0$

$\Sigma F_x = ma_x$

$F_A \cos\theta - F_K = ma_x$

$\Sigma F_{Ax} = ma_x$

$F_A \cos\theta - F_K = ma_x$

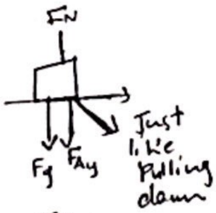
since $f_K = \mu_K \cdot F_N$

we need to find how each state (Push or Pull) affect the normal force bc it's the one in control of f_K

$\Sigma F_y = 0$

Push: $F_N - F_{Ay} - mg = 0$

$F_N = mg + F_A \sin\theta$



Pull : $F_N + F_{Ay} - mg = 0$

$F_N = mg - F_A \sin\theta$

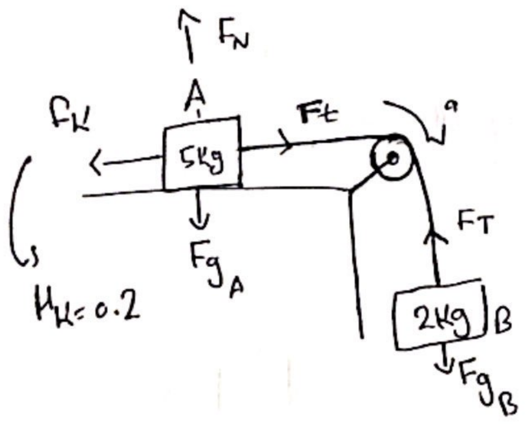
and $f_s = \mu_s \cdot F_N$

the higher the F_N the more friction

- during the push motion F_N is > the pull

so you exert less friction on Pull thus you will use less force example :

Two boxes connected by a cord around a Pully



they have to have same Tension and acceleration so calculate them

A

$\Sigma F_y = 0$

$F_N = Fg_A$

$F_N = 50N$

$f_K = \mu_K F_N$

$= 50 \times \frac{2}{10} = 10N$

B

$\Sigma F_y = m a_y$

$F_T - Fg_B = m_B a_y$

$F_T - 20N = -m_B a$

$F_T = -m_B a + 20$

$\Sigma F_x = m a_x$

$F_T - f_K = m_A a_x$

~~$m_B a + 20 - 10 = m_A a$~~

~~$2a - 10 = 5a$~~

~~$3a = 30$~~

$20 - m_B a - 10 = 5a$

$10 = 7a$

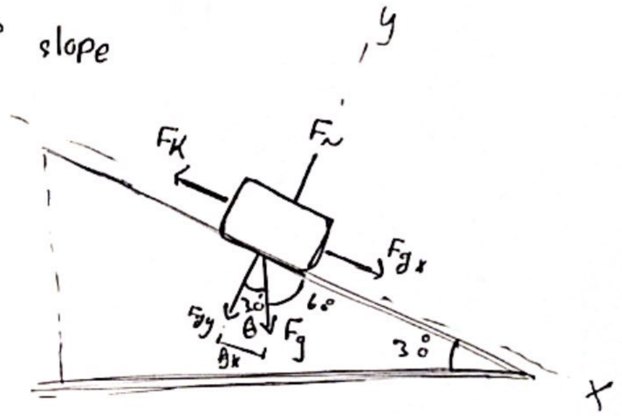
$a = 1.42 \ m/s^2$

$F_T = -2(1.42) + 20$

$F_T = 14.16 \ N$

Example 4-21

a skier is descending a 30° slope
-if $\mu_k = 0.1$
find a (acceleration)



$F_{gx} = F_g \sin\theta$

$F_{gy} = F_g \cos\theta$

$f_k = \mu_k F_N$

$= \mu_k (F_g \cos\theta)$

$\Sigma F_y = 0$

$F_N - F_{gy} = 0$

$F_N - F_g \cos\theta = 0$

$$\boxed{F_N = F_g \cos\theta}$$

$\Sigma F_x = m a_x$

$F_{gx} - F_k = m a_x$

$F_g \sin\theta - F_k = m a_x$

$F_g \sin\theta - \mu_k (F_g \cos\theta) = m a_x$

$\frac{mg \sin\theta}{m} - \frac{\mu_k mg \cos\theta}{m} = \frac{m a_x}{m}$

$g \sin\theta - g\mu_k \cos\theta = a_x$

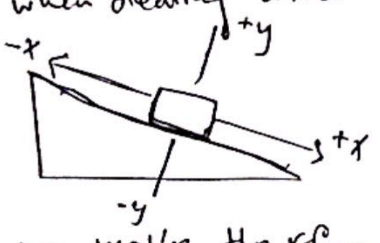
$5 - 0.1(10)\left(\frac{\sqrt{3}}{2}\right) = a_x$

$$\boxed{a_x = \frac{10 - \sqrt{3}}{2}}$$

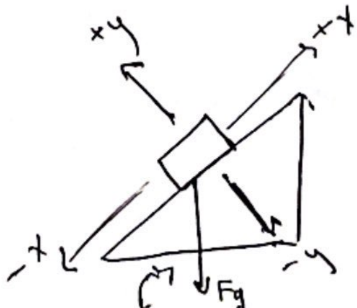
m/s²

---

when dealing with inclines



we make the reference x and y axis with it



★ Fg is always downward (not with the axis)